

A Dipole interpretation of the $\nu = 1/2$ state

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Abstract

We consider the problem of Bosonic particles interacting repulsively in a strong magnetic field at the filling factor $\nu = 1$. We project the system in the Lowest Landau Level and set up a formalism to map the dynamics into an interacting Fermion system. Within a mean field approximation we find that the composite Fermions behave as a gas of neutral dipoles and we expect that the low energy limit also describes the physical $\nu = 1/2$ Fermionic state.

1 Introduction

There has been recently a renewed interest in the quantum Hall effect when the filling factor is a fraction with an even denominator. Willets and his collaborators [5] have observed an anomalous behavior in the surface acoustic wave propagations near $\nu = 1/2$ and $\nu = 1/4$. A remarkable outcome of their experiments is that they probe a longitudinal conductivity $\sigma_{xx}(q, \omega)$ increasing linearly with the wave vector q . Halperin, Lee and Read [6] have suggested that the system exhibits a Fermi liquid behavior at this particular value. They have developed a formalism based on the Chern-Simon theory which provides an explanation for the experimental observations. Subsequently, several studies have developed and improved the predictions of the Chern-Simon theory [6] [11] [12] [13] [14]. Another approach followed by Rezayi and Read [15] and Haldane et al [20] consists in obtaining trial wave functions to study numerically the properties of the system at this filling factor. In these studies the cyclotron frequency is supposed to be sufficiently large so that the only relevant excitations are confined to the lowest Landau level. The trial wave functions can be compared to the exact ground state and the overlap between the two turns out to be extremely good [15]. In these studies the effective mass m^* which defines the Fermi velocity is generated dynamically by the interactions. This paper introduces a microscopic model closely related to these trial wave functions. We have considered toy model of Bosonic particles interacting repulsively in a magnetic field at a filling factor $\nu = 1$. Although it may at first look different, the problem of formation of a Fermi sea is essentially the same as in the $\nu = 1/2$ case. If one applies the analyses of the composite Fermions [2] or the Chern-Simon approach to such a system, one is led to the same picture of Fermi sea formation as in the $\nu = 1/2$ case.

Halperin Lee and Read use the composite Fermions arguments to motivate the formation of a Fermi sea in the $\nu = 1/2$ case. They attach two magnetic fluxes to an electron in order to cancel the magnetic field seen by the electron in the mean field approximation. This flux attachment does not modify the statistics of the electrons and if one ignores fluctuations one is led to a system of spinless Fermions in a zero magnetic field. In the case of Bosons at $\nu = 1$, we can proceed similarly by attaching one flux unit to each particle so as to cancel the exterior magnetic field. In this process the statistics is changed from

Bosons to Fermions and a Fermi liquid is expected to form.

Read has interpreted the fluxes attached to the electron as physical vortices bound to it [7](see also [8] and [9]). We claim that his proposal differs from the mean field interpretation for the following reason. The mean field treats the composite electron as a charged particle which couples to the electro-magnetic field. The vortices carry a charge equal to minus one half of that of the electron so that the bound state is as a neutral particle which propagates in a constant charge background. In this case the response to an external field depends on the dipole structure of the composite object. We are led to this picture in the $\nu = 1$ case. The main simplification is that there is a single vortex coupled to the Boson, this vortex is a Fermion carrying the opposite charge and we can use a second quantized formalism to analyse the model. The bound state is then a dipole [18] [17] [16].

Our approach is mainly motivated by the trial wave functions of [15] [20]. If the particles were distinguishable the Laughlin wave function [1] would be the best ground state but it gives the particles the wrong statistics. One corrects for this by multiplying the trial wave functions by a Slater determinant of plane waves and project the product into the lowest Landau level (LLL). The effect of the projection is to replace the coordinate in the plane waves by operators[15] which displace the particles from their original position. Here we advocate that the charge fluctuations induced by this displacement are the fundamental excitations.

The next section presents the microscopic model and analyze its phenomenological consequences.

2 The Microscopic Model

2.1 motivation

Consider N particles of identical charge interacting with a repulsive force in a domain of area Ω thread by a magnetic field B . B is chosen so that the flux per unit area is equal to one. The magnetic length $l = \sqrt{\hbar c / eB}$ is such that $\Omega = 2\pi l^2 N$. We assume that the cyclotron frequency is large compared to the interaction so that the dynamic can be restricted to the Lowest Landau Level. The one body Hamiltonian has N degenerate eigenstates, thus in the case where the particles are

Fermions the only accessible state is given by the Slater determinant of the one body wave functions. Suppose the particles are divided into two sets which differ only by the statistics they obey. The first set contains N_1 Fermions and the second set contains N_2 Bosons the sum $N_1 + N_2 = N$ being kept fixed so that the filling factor remains equal to one and the interaction are the same between all the particles. The simplest case consists of 1 Boson interacting with $N - 1$ Fermions. By performing a particle hole transformation on the Fermions we can equivalently regard this as a Boson interacting with a hole, a problem studied by Kallin and Halperin [16].

In the Landau gauge one body Hamiltonian is proportional to:

$$H = p_x^2 + (p_y - x/l^2)^2 \quad (1)$$

This Hamiltonian commutes with the two guiding center coordinates $R_y = l^2 p_x - y$ and $R_x = l^2 p_y$ which do not commute with each other $[R_x, R_y] = i l^2$. In the particle hole case the Hamiltonian of the pair is the sum of two one body Hamiltonians where we change the sign of the the potential vector for the hole. Since the particles have exactly opposite charges, the guiding center coordinates of the pair $R_{y1} + R_{y2} = l^2 p_{x1} + l^2 p_{x2} - (y_1 - y_2) = l^2 p_x$ and $R_{x1} + R_{x2} = l^2 p_{y1} + l^2 p_{y2} = l^2 p_y$ commute with each other and can be diagonalized simultaneously with H . The wave function which diagonalizes this generalized momentum describes a dipole propagating freely with its dipole vector $l^2 (p_y, -p_x)$ perpendicular to the momentum (p_x, p_y) . The potential interaction commutes with the momentum p_x, p_y so that these wave functions are eigenstates of the total projected Hamiltonian. When the interaction between the particles is repulsive these wave functions describe bound states of size comparable to the magnetic length with a mass of the order of $V(l)$.

In the general case, if N_2/N is small compared to 1, it is legitimate to subdivide the particles and the holes into pairs so as to include the interaction in each pair in the one body Hamiltonian H_0 and treat residual interaction between the different pairs as a perturbation. When this ratio is equal to one ($N = N_2$) it is more difficult to argue that the low density approximation is valid. Nevertheless, because the bound states are Fermions and if we assume that only the quasiparticles at the Fermi surface participate to the dynamics, this approximation still makes sense. In the next section we set up the formalism based on this general philosophy to map the Bosonic

problem into a Fermionic one.

2.2 General Formalism

We consider the case of a finite geometry such that the degeneracy of the LLL is equal to the number of bosons N . A basis of LLL orbitals is indexed by $i, 1 \leq i \leq N$. To each orbital we associate the canonical Bosonic creation operator a_i^+ which creates a state in this orbital. The Hilbert state of the $\nu = 1$ Bosons is generated by the states where N creation operators act upon the vacuum state $|0\rangle$ defined by $a_i|0\rangle = 0$.

One can map the Bosonic space into a subspace of a Fermionic space proceeding as follows: To the a_i^+, a_i we adjoin a set of canonical Fermionic operators f_i^+, f_i also labeled by the LLL orbitals and consider the vacuum $|\Omega\rangle$ obtained by filling the Fermionic orbitals $f_i^+|\Omega\rangle = b_i|\Omega\rangle = 0$. The Bosonic Hilbert space is recovered upon acting on $|\Omega\rangle$ with N pair creation operators $b_i^+ f_j$. The idea (called the method of images in other contexts [4]) is to substitute a creation operators χ_{ij}^+ for the pair $b_i^+ f_j$. We thus consider a set of operators defined by:

$$\begin{aligned}\{\chi_{ij}, \chi_{kl}\} &= \{\chi_{ij}^+, \chi_{kl}^+\} = 0 \\ \{\chi_{ij}^+, \chi_{kl}\} &= \delta_{il} \delta_{jk}\end{aligned}\quad (2)$$

The Fermionic Hilbert space is obtained upon acting with $N \chi_{ij}^+$ on the vacuum $|\Omega'\rangle$ annihilated by the χ_{ij} . This description of the original Bosonic space is still overcomplete since the pairing between Bosons and Fermions is arbitrary in the definition of the pairs χ_{ij} . To recover the physical space, we must project the Hilbert space generated by the χ_{ij}^+ onto the sub-space antisymmetric under the permutations of the Fermionic indices j :

$$b_{i_1}^+ \dots b_{i_n}^+ |\Omega\rangle = 1/N! \sum_{p \in S_N} (-)^p \chi_{i_1 p_1}^+ \dots \chi_{i_N p_N}^+ |\Omega'\rangle \quad (3)$$

Next we identify the observables in both representations as follow:

$$\begin{pmatrix} \rho_{ij}^b & A_{il}^+ \\ A_{kj} & \rho_{kl}^f \end{pmatrix} = \begin{pmatrix} b_i^+ b_j & b_i^+ f_l \\ f_k^+ b_j & f_k^+ f_l \end{pmatrix} = \begin{pmatrix} (\chi^+ \chi)_{ij} & (\chi^+ \sqrt{1+:\chi\chi^+:})_{il} \\ (\sqrt{1+:\chi\chi^+:})_{kj} & (1+:\chi\chi^+:)_{kl} \end{pmatrix} \quad (4)$$

Where the normal ordering refers to the vacuum.

To establish (4), we must verify that both sets of operators obey the same $U(N|N)$ algebra and that the representations obtained by acting with them upon the respective vacua are equivalent. It is straightforward to verify that the diagonal blocks ρ^b, ρ^f obey the same commutation relations. It is less obvious that the relation:

$$\{A_{kj}^+, A_{il}\} = \delta_{ji}\rho_{kl}^f + \delta_{kl}\rho_{ij}^b \quad (5)$$

are satisfied in the χ representation and can be shown along the lines of [10]¹. The operators A_{il}^+ in the upper right corner are the pair creation operators used to generate the Hilbert space upon acting on the vacuum. In what follow, we shall keep only the first term in the expansion of the square root and simply replace them by χ_{il}^+ . This amounts to disregard the projection in (3).

The respective vacua $|\Omega >, |\Omega' >$ are both annihilated by the down left block matrices A_{kj} , thus defining highest weight representations of $U(N|N)$. The equivalence of the representations follows from action of the diagonal blocks $\rho^{b,f}$ on the vacuum:

$$\rho_{ij}^b |\Omega > = 0, \quad \rho_{kl}^f |\Omega > = \delta_{kl} |\Omega > \quad (6)$$

The original Boson dynamics can be expressed in terms of the density operators ρ_{ij}^b which obey the $U(N)$ algebra. To describe the dynamics one possibility would be to use the expression of these operators terms of χ . Here, we treat the Fermions as real particles which see the external field in the same way as the Bosons. This amounts to replace the density operators by

$$\rho_{ij} =: \rho_{ij}^b + \rho_{ij}^f :=: \{\chi^+, \chi\}_{ij} : \quad (7)$$

Since there is no Fermion when $\nu = 1$ ρ_{ij}^f is essentially equal to zero. This modification nevertheless affects the way we approximate the system. In particular, the dynamics is now well defined when the number of χ^+ operators which act on the vacuum is not equal to N and this allows us to vary the density of pairs arbitrarily.

¹These authors consider the more general case where the particles carry a flavor index ($b_j \rightarrow b_j^a$) taking n values which is summed in the upper matrix of (4). In this case 1 must be replaced by n in the lower matrix of (4).

To recover the real space description let us for concreteness consider the case of a rectangular box of size L_x, L_y with $L_x L_y = 2\pi l^2 N$. We set $z = (x + iy)/L_y$ and $\tau = L_x/L_y$. In these notations the LLL orbitals wave functions are given by:

$$\langle \vec{x} | j \rangle = \frac{1}{\sqrt{\pi L_y}} e^{-\pi x^2/\tau} \theta_j(z) \quad (8)$$

where $1 \leq j \leq N$ and θ_j is the theta function defined as:

$$\theta_j(z, \tau) = \sum_{n \in \mathbb{Z}} \exp(-\pi(j + nN)^2 \tau + 2\pi(j + Nn)z) \quad (9)$$

Except for a common factor these wave functions depend on x, y only through the variable z and a family of coherent states $|z\rangle$ can be defined [3] such that $\langle z | i \rangle = \langle \vec{x} | i \rangle$. Suppose that the LLL particles interact with a scalar potential $V(\vec{x} - \vec{y})$. After projection the Hamiltonian takes the form:

$$H = 1/2 \int \rho(\vec{x}) V(\vec{x} - \vec{y}) \rho(\vec{y}) d^2x d^2y \quad (10)$$

where $\rho(\vec{x})$ is the projected density operator

$$\rho(\vec{x}) = \langle z | \hat{\rho} | z \rangle = \sum_{i,j} \langle \vec{x} | i \rangle \rho_{ij} \langle j | \vec{x} \rangle \quad (11)$$

The projection relates a field $\rho(\vec{x})$ to a matrix $\hat{\rho}$ and more generically, the transformation which associates the function $\rho(x) = \langle z | \hat{\rho} | z \rangle$ to the matrix ρ_{ij} is called its P-symbol in [3]. Since translations act naturally on $\rho(x)$ and there are N^2 matrix elements² we can decompose $\rho(x)$ onto N^2 plane waves $\rho(x) = \sum_{\vec{k}} e^{i\vec{k}x} \rho_{\vec{k}}$ with $k_i = 2\pi n_i/L$, $0 \leq n_1, n_2 \leq N-1$. By the inverse transformation, $\rho(x) = \langle z | \hat{\rho} | z \rangle$ where $\hat{\rho} = 2\pi l^2 \sum_{\vec{k}} e^{k^2 l^2/4} \hat{e}^{\vec{k}} \rho_{\vec{k}}$ and the matrices $\hat{e}^{\vec{k}}$ obey the magnetic translation algebra ([19]):

$$\hat{e}^{\vec{k}} \hat{e}^{\vec{q}} = e^{il^2(\vec{k} \times \vec{q})/2} \hat{e}^{\vec{k}+\vec{q}} \quad (12)$$

This defines a matrix product on functions which we denote by \star to distinguish it from the ordinary product.

²since there are only N^2 Fourier modes the system is in fact defined on a square lattice with a lattice cut-off a equal to $\sqrt{2\pi}l/\sqrt{N}$. A difficulty is that we have to deal with three length scales $a \ll l \ll \xi$ where ξ is the physically relevant scale.

Let $\Psi(x) = \sqrt{2\pi l^2} < z | \hat{\chi} | z >$ denote the space dependent field associated to the matrix field χ_{ij} . The commutation relations (2) imply the following decomposition for $\Psi^+(x)$:

$$\Psi^+(x) = 1/L^2 \sum_k e^{-k^2/4} e^{ikx} c_k^+ \quad (13)$$

where c_k^+, c_k are canonical Fermionic operators.

In these notations the density (7) is given by:

$$\rho(\vec{x}) = \frac{1}{2\pi l^2} : \{ \Psi^+ \star, \Psi \} (x) : \approx l \vec{\nabla} \times \Psi^+ i l \vec{\nabla} \Psi(x) \quad (14)$$

The anticommutator originates from the fact that we add the two contributions ρ^b and ρ^f treating the pairs as composite particles. As a result, the dominant term in a gradient expansion is the right-hand side of this equality.

The Hamiltonian (10) can be expressed in terms of these operators and the most relevant contributions around a Fermi surface is given by:

$$H = \int d^2x \Psi^+ (-\Delta/2m^* + \mu) \Psi(\vec{x}) + \int d^2x \int d^2y \\ (\Psi^+ i l \vec{\nabla} \Psi(\vec{x}) \times l \vec{\nabla}_x) (\Psi^+ i l \vec{\nabla} \Psi(\vec{y}) \times l \vec{\nabla}_y) V(\vec{x} - \vec{y}) \quad (15)$$

where the effective mass m^* is of the order of magnitude of $V(l)$. Although our derivation is only valid for the $\nu = 1$ case, it is tempting to assume that the low energy limit of this Hamiltonian also describes the physical situation $\nu = 1/2$. In this case, the chemical potential μ must be adjusted so that the density is equal to $\nu/2\pi l^2$ which imply that the Fermi momentum $k_F = \sqrt{2\nu}/l$. Recently, Shankar and Murthy [21] have derived the same Hamiltonian in the $\nu = 1/2$ case using a different approach.

The interaction has no Gallilean invariance which is not surprising in the presence of a magnetic field. If $V(r)$ behaves as r^{-1} at large distance, the induced dipole potential behaves as r^{-3} and we expect no infrared singularity. The system becomes essentially equivalent to a Fermi liquid with short range interaction.

Consider now the linear response to a scalar field $\Phi(\vec{x}, t)$. In the long wavelength limit the interacting Hamiltonian is given by:

$$H_i = \int d^2x \ l \vec{\nabla} \Phi(\vec{x}, t) \times \Psi^+ i l \vec{\nabla} \Psi(\vec{x}) \quad (16)$$

As a consistency check we can couple the system to a constant electric field $\Phi(\vec{x}) = e\vec{E} \cdot \vec{x}$. In this case the interacting Hamiltonian $H_i = e\vec{E} \times \vec{K}$ where \vec{K} is the total momentum. Its only effect is to give an additional speed $e|E|$ in the direction perpendicular to \vec{E} to each quasiparticle. Thus one recovers the value of the transverse conductivity $\sigma_{xy} = \nu e^2/2\pi\hbar$. More precisely, the vacuum is charged and responsible for the current while the quasiparticles are neutral and carried by the vacuum.

Using the transport equation in presence of the interaction (16), one obtains the static response function and the dynamical form factor:

$$\begin{aligned}\chi(\vec{q}, 0) &= -(l^2 q k_f)^2 \nu(0)/2(1 + F_1) \\ S(\vec{q}, \omega) &= S^0(\vec{q}, \omega)(l^2 k_f q/1 + F_1)^2\end{aligned}\tag{17}$$

$\nu(0) = m^* \Omega/2\pi$ is the density of states on the Fermi surface. $S^0(\vec{q}, \omega)$ is the free fermion form factor and F_1 is the first Landau parameter (F_0 is not relevant in this theory). The essential difference with the Fermi liquid results are the factors proportional to $(lq)^2$ which damp the effect of the external field at low q and originate from the fact that dipoles couple weakly to an external potential.

2.3 Conclusion

We have introduced a microscopic model to analyze the problem of Bosonic particles in a strong magnetic field at $\nu = 1$. We have generalized the model so as vary the Fermi momentum p_f and to study it in a mean field approximation.

The present model gives a description in agreement with the dipole picture introduced by N.Read [7]. The main conclusion of our study is that the system behaves essentially as a gas of Fermionic dipoles with a dipole vector perpendicular to their momentum. As a result, the interactions are screened and the gas behaves in many respect as a neutral Fermi liquid (the same conclusion is reached in [9]). The main consequence is that the linear response quantities get renormalized by a factor $(lq)^2$ at low momentum transfer q . The Landau theory only relies on the hypotheses that the quasiparticles are dipoles and should therefore also be valid in the $\nu = 1/2$ case.

The model differs from the Chern-Simon theory by the fact that the dynamics is projected into the Lowest Landau Level and the effective

mass depends only on the interactions. This model does not seem to predict a divergence of the effective mass.

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